# HYDRODYNAMIC INTERACTION BETWEEN EXPLOSIVE FUSE CHARGES

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## INTRODUCTION

The interaction problem for explosive fuse charges is studied within a formulation proposed by M. A. Lavrent'ev. The case of two concentrated changes and that of a uniform number of evenly distributed surface charges are considered. It is shown that the area of the ejection cone decreases in the interaction. Highly distributed explosive charges, called fuse charges, were first successfully applied for different types of excavation and earth-moving operations in the Ukraine in the 1940's and 1950's under the direction of M. A. Lavrent'ev. Most of these operations, such as the digging of wells and the laying of drainage channels, were carried out at that time by N. M. Sytyi. A large number of concentrated charges were used (either connected by a detonation fuse or having a capsule inserted into each charge, after which it was exploded) in order to construct a trench in the soil with high length–width ratio or, for example, to create a shock wave that would have a cylindrical front at significant distances and for a long duration. M. A. Lavrent'ev and N. M. Sytyi proposed that a narrow ditch be cut in the soil (if we are speaking of a trench) in which the explosive charge is laid without exploding and that the explosion be carried out using a single detonator. Fuse charges have now become extraordinarily widespread. An approximation method of calculating the dynamic stress and velocity fields, based on a model of an ideal incompressible fluid, is often used to theoretically estimate the effect of an underground charge. Lavrent'ev [1] solved the production and effect problem for a blast from a hollow charge within the framework of this model. Steady fluid flow was considered here. A model of an ideal incompressible fluid has been used [2] to calculate the cone dimensions for an underground ejection explosion, a pulsed formulation of the problem was applied, and the complementary bearing power of the spoil (or critical velocity) was introduced. The ideas of Lavrent'ev were subsequently developed in works on the shape of the ejection cone [3-5], on the principles of an absolute directed explosion [6-7], on the disruptive effect of an explosion [8, 9], and on a principle of a uniform grinding for rock [10].

A pulsed formulation of the hydrodynamical problems is usually used in solving explosion problems [11]. Suppose we have a region D with boundary Γ filled with an ideal incompressible fluid. A pressure p (Q, t), where $Q \in \Gamma$ acting for a brief period of time $\tau$ is defined at points of the boundary. It is required that the field of pressures p (M, t) and velocities $\vec{v}$(M, t), where $M \in D$, be determined in D. To solve this problem, we introduce the pressure pulse

$$P(M) = \int_0^\tau p(M, t)\, dt, \qquad (0.1)$$

with the condition [11] that pressure p (Q, t) is short-term, $\vec{v} = -(1/\rho)\,\mathrm{grad}\,P(M)$.

The incompressibility condition $\mathrm{div}\,\vec{v} = 0$ implies that the pressure pulse P (M) satisfies the Laplace equation $\Delta P = 0$. Thus it is necessary to solve the Dirichlet problem for a Laplace equation under the boundary condition (0.1). The pressure and velocity calculated using this method are independent of time, which corresponds to the concept that the process in which the pressure and velocity fields are established at the initial stage of motion arising in the explosion is short-term.

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From the physical point of view, the creation of an initial velocity field is the result of propagation, reflection, and interaction between the stress waves in the medium surrounding the explosive. Therefore, the scheme of an ideal incompressible fluid is refined in the most desirable way by establishing its relation to exact solutions of wave-propagation problems in a compressible medium.

It has been shown [12, 13] in an acoustic approximation that the model of an ideal incompressible fluid "can be interpreted as the integral asymptotic (as $t \to \infty$) of the compressible-medium problem." There exist two varieties of this model. In one model (the fluid model) [2] the soil is considered as an ideal incompressible fluid throughout the entire region it occupies. In the second model (solid–fluid model), the soil is described by equations for an ideal incompressible fluid only in some region near the charge. Outside the region, the soil behaves as an absolutely rigid body, while the boundary separating the fluid is a solid wall, found by setting the velocity modulus on it equal to the critical magnitude $c*$.

Two problems in the solid–fluid model for the interaction of explosive charges were considered as an example. They are of definite interest from the practical point of view, since the explosion of systems of charges is encountered quite often. At the same time, it is unclear what it is that an interaction of charges leads to, that is, how the dimensions of the ejection region vary.

## 1. Interaction Problem for an Infinite System of Plane Surface Charges

Suppose the region D is in the form of a lower half-plane, and let an infinite series of plane charges of length $2l$ be situated on the surface $y = 0$, $2m$ units apart. The effect of each charge is determined by the pulse pressure P, since a potential $\varphi = -\varphi_0 = -P/\rho$ is defined on each segment.

Because of symmetry, we will limit ourselves to considering the region formed by two vertical lines passing through the midpoint of one charge and the midpoint of the distance between them. Let us introduce the dimensionless variables $\bar{w} = w/\varphi_0$ and $\bar{z} = zc*/\varphi_0$ (superscripts will be henceforth omitted). The flow region in the physical plane is depicted in Fig. 1a. On the segment CD $\varphi = 0$ (free boundary). The unknown boundary AB is a segment of the streamline EABC on which the stream function $\psi = 0$. Moreover, the modulus of velocity is constant and equal to one on the boundary AB. It is necessary to find from these data an analytic function $w(z) = \varphi + i\psi$ that is the complex flow potential and also determines the lines AB. The flow region is in the form of a half-band (Fig. 1b) in the plane of the complex potential.

Let us introduce the function $\xi = \ln dz/dw$. The flow region in the plane $\xi$ is also in the form of a half-band (Fig. 1c). The problem will reduce to the conformal mapping of the half-bands represented in Fig. 1b and c with the indicated correspondence of points, obtained by means of an intermediate mapping on the half-plane, and has the form

$$\cos \pi w = \frac{e - c - 2i\,\mathrm{sh}\,\xi}{e + c}, \tag{1.1}$$

where c and e are the parameters of the problem,

$$c = \frac{1}{2}\left(v + \frac{1}{v}\right), \quad e = \frac{1}{2}\left(u + \frac{1}{u}\right)$$

(v is the velocity at the point C and u, the velocity at the point E). We obtain from Eq. (1.1) the ordinary differential equation

$$\frac{2}{e+c}\frac{dz}{dw} = i\left[\cos \pi w - \frac{e-c}{e+c}\right] + \sqrt{(b - \cos \pi w)(a + \cos \pi w)},$$

$$b = \frac{2 + e - c}{e + c}, \quad a = \frac{2 - e + c}{e + c},$$

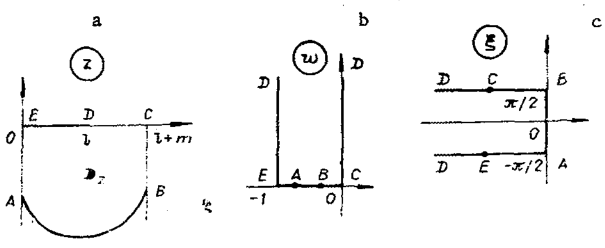


Fig. 1

whose integration yields the desired solution. The integral along the segments ED and DC indicates a relation between the parameters c and d and the initial data $l$ and m, and, along the curve AB, the shape of the cone boundary. Figure 1a depicts the shape of

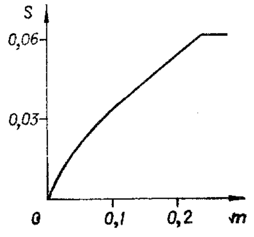


Fig. 2

the ejection cone at $l \approx 0.1$ and $m \approx 0.095$. When $c = 1$ ($v = 1$, points B and C coinciding), the charges do not interact, and our solution coincides with a previously found solution [3].

When $e = 1$ ($u = 1$) points E and A coincide, and only part (depending on the size of c) of the charge will affect the ejection. It is of interest to note that if flow is reversed (i.e., if we assume that the segment DC is the charge and that AD is the free surface), our solution again coincides with the previously found solution [3].

The area S of the ejection cones was calculated for different distances between the charges when $l = 0.1$ (when $m > 0.244$, the charges do not interact) in order to estimate the efficiency of the operation of the system of charges. This dependence is depicted in Fig. 2. The interaction of charges under our formulation decreases the cone area. One feature of the problem is that as $m \to 0$, the entire system of charges reduces to a single infinitely distant charge, and, in general, no ejection occurs. Expressed otherwise, it is found that if we take two charges in place of an infinite system, the ejection cone does not vanish when they have completely approached each other.

Let us consider the interaction of two concentrated surface charges whose effect in the hydrodynamic model is described by dipoles with moment M. The size of M for a surface charge is determined by the formula $M = \varphi l$, where $\varphi$ is the potential arising on the charge line and $l$ is charge length. Since $\varphi$ is proportional to the charge thickness, we find that M is proportional to the charge energy. It is known from dimensionality theory that the characteristic dimension of a cone from such a charge is proportional to $\sqrt{M/c_*}$, while the area is proportional to the area of the explosives. Consequently, the area of the cone is equal to the total area of the noninteracting charges as the two surface charges merge.

Let us consider the problem in more detail in order to determine the influence of interaction in intermediate cases.

## 2. Explosion of Two Surface Charges

The general form of the flow is depicted in Fig. 3a. Because of the symmetry of the problem, we will limit ourselves to the right half-plane. A charge situated at point B ($x = L$, $y = 0$) will be said to be a dipole with moment M. It is required that the complex flow potential $w(z) = \varphi + i\psi$ and the unknown part of the boundary DC be found in the region $D_z$ from the boundary conditions $\varphi = 0$ on the free surface AC, $\psi = 0$ in ADC (AD is the streamline, by symmetry), and $|dw/dz| = c_*$ on the unknown part DC. We introduce the dimensionless variables

$$\bar{w} = \frac{w}{\sqrt{Mc_*}}, \quad \bar{z} = z\sqrt{\frac{c_*}{M}}.$$

Then $|d\bar{w}/d\bar{z}| = 1$ on DC and $\bar{w} \approx i/(z - \bar{L})$ in a neighborhood of B. The bars will henceforth be omitted over dimensionless variables.

We will use the method of singularities [14, 15] for the solution. We introduce a parametric complex variable t and form a flow region $D_z$ and a quadrant of the unit circle ADC of the plane t with the point correspondence $t_A = 0$, $t_D = 1$, and $t_C = i$ (Fig. 3b). Here the point $t_B = ih$ (h is determined in the course of solving the problem) corresponds to B. The flow region in the plane w is depicted in Fig. 3c. Let us construct the function $w(t)$. In a neighborhood of $B(t = ih)$ $-\infty \le \psi \le \infty$, i.e., there exists a flow with an infinite flow rate. The independent variables vary by the same magnitude as this point is circuited in the t and w planes. Consequently, a first-order pole corresponds to this point for the function $w(t)$. When $t = 0$ and $t = i$ (points A and C), $w(t)$ has first-order zeros (the conformal property is not violated at these points and $dw/dt \neq 0$). The conformal property is violated at the points F ($t = r$) and D ($t = 1$), so that $dw/dt = 0$. Since $\psi = 0$ on ADC and $\varphi = 0$ on ABC, w may be continued, by symmetry, to the entire complex plane with poles at $t = \pm ih$ and $\pm i/h$, and zeros at the points $t = 0$ and $t = \pm i$. It is rational except for the other singular points and has the form

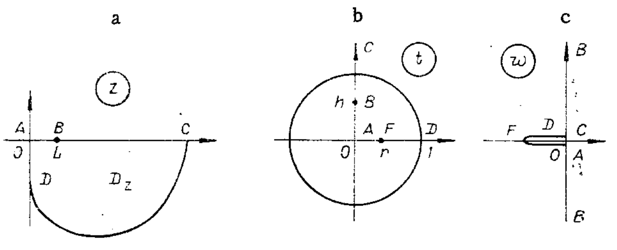


Fig. 3

$$w = -A\frac{t(t^2+1)}{(t^2+h^2)(1+t^2h^2)}, \quad (2.1)$$

where A is a real constant.

We find by direct computation that

$$\frac{dw}{dt} = \frac{Ah^2}{r^2}\frac{(1-t^2)(t^2-r^2)(1-t^2r^2)}{(t^2+h^2)^2(1+t^2h^2)^2}, \quad (2.2)$$

where

$$r + \frac{1}{r} = \frac{1}{h} - h$$

or

$$r = \frac{\frac{1}{h} - h - \sqrt{\left(\frac{1}{h} - h\right)^2 - 4}}{2}, \quad (2.3)$$

when $h < h_1 = \sqrt{2} - 1$, where r is a real number, $0 < r < 1$. When $h = h_1$ and $r = 1$,

$$r = \frac{\frac{1}{h} - h + i\sqrt{4 - \left(\frac{1}{h} - h\right)^2}}{2},$$

as $h > h_1$ increases further, where r is a complex number, $|r| = 1$. The point F travels along the arc CD in Fig. 3b correspondingly when $h > h_1$ and the point D travels along the lower coast of the section FA in Fig. 3c.

Let us now calculate the function $dw/dz \equiv \omega(t)$. At the point $t = r$

$$\frac{dw}{dt} = \frac{dw}{dz}\frac{dz}{dt} = 0.$$

If r is a real number, $dz/dt \neq 0$, since the conformal property is not violated on AD under the mapping $z \to t$, and, consequently, $dw/dz = 0$. But if r is a complex number, $|dw/dt| = 1$ and $dz/dt = 0$. We will therefore consider each case separately.

1) $h < h_1$, r a real number. In this case the exterior of a unit semicircle and the section along the imaginary semiaxis (Fig. 4a) is the flow region in the $\omega$ plane. As the point B in the t plane is circuited, the independent variable varies by $\pi$, and varies by $2\pi$ as a circuit is carried out in the $\omega$ plane, so that the function $\omega$ has a second-order pole at the point B ($t = ih$). At the point F ($t = r$) $\omega$ has a first-order zero. A line segment in the $\omega$ plane corresponds to the line segment AFD in the t plane, so that, by symmetry, $\omega$ can be continued through AFD and, analogously, through ABC, and we may establish that it has a second-order pole at the point $t = -ih$ and a first-order zero at the point $t = -r$. The function $\omega$ is now defined in the unit circle and when $|t| = 1$, $|\omega| = 1$. By symmetry, it can be continued to the entire complex plane by means of inversion. Consequently, it has second-order zeros at the points $\pm i/h$ and poles at the points $\pm 1/r$. Direct verification shows that

$$\omega = i\frac{(1+t^2h^2)^2(t^2-r^2)}{(t^2+h^2)^2(1-t^2r^2)} \quad (2.4)$$

is the desired function.

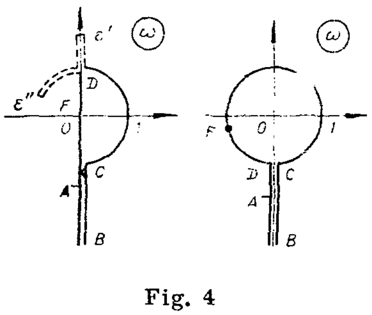

Fig. 4

We note that the flow region in the $\omega$ plane has a groove $D\mathscr{E}'F$ or $C\mathscr{E}''D$ as a function of h. For small h, this is the groove $D\mathscr{E}'F$ which decreases with increasing h and vanishes when $h = h_2$. A groove $C\mathscr{E}''D$ then arises and as $h \to h_1$ ($r \to 1$), the independent variable for the point $\mathscr{E}''$ tends to the value $3\pi/2$ and the flow region qualitatively

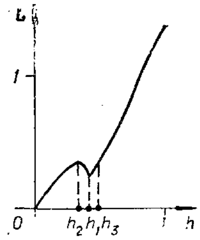


Fig. 5

varies (Fig. 4b). The conformal property of the mapping is violated at the point $\mathscr{E}'$ (or $\mathscr{E}''$) and $d\omega/dt$ has a first-order zero. When $h = h_2$ (no grooves), the conformal property is violated at the point D. In this case $d\omega/dt$ has a second-order zero, since $\omega(t)$ has the expansion $\omega(t) - i \approx (t-1)^3$ at this point. We will find the value of $h_2$ and the points $t_{\mathscr{E}'}$ and $t_{\mathscr{E}''}$. The solution of the equation $d\omega/dt = 0$ reduces to the solution of the biquadratic equation

$$t^4 - 2bt^2 + 1 = 0,$$

where

$$b = \frac{\left(\frac{1}{h} + h\right)\left(\frac{1}{r^2} + r^2\right) - \frac{1}{2}\left(\frac{1}{h^2} + h^2\right)\left(\frac{1}{r} - r\right)}{\frac{1}{r} - r + 2\left(\frac{1}{h} + h\right)}.$$

The desired solution has the form

$$t_{\mathscr{E}} = \sqrt{b - \sqrt{b^2 - 1}}.$$

We prove that $b > 0$ proceeding on the basis of $h < \sqrt{r-1}$; consequently, flow from the point $\mathscr{E}'$ or $\mathscr{E}''$ exists as a function of the condition $b > 1$ or $b < 1$. Solving the equation $b = 1$, we find that $h_2$ is the solution of the equation

$$\frac{1}{r} - r = \frac{1}{2}\left(\frac{1}{h} + h\right)$$

and, consequently, $h_2 = (\sqrt{8} - \sqrt{5})\sqrt{3} \approx 0.342$. Thus $t_{\mathscr{E}}$ is a real variable, $t_{\mathscr{E}} = t_{\mathscr{E}'}$, and belongs to FD when $0 < h < h_2$; when $h_2 < h < h_r$ $t_{\mathscr{E}}$ is a complex variable, $t_{\mathscr{E}} = t_{\mathscr{E}''} = e^{i\theta}$, and belongs to DC, where $0 < \theta < \pi/2$. The presence of the groove $\mathscr{E}$ means that the modulus of velocity rose on the segment FD from zero to some magnitude greater than 1 (the point $\mathscr{E}'$), and then decreases to the point D (cf. Fig. 4a). The groove $\mathscr{E}''$ indicates that the independent variable for the velocity is equal to $-\pi/2$ at D, and does not grow monotonically to $\pi/2$ at C, but first decreases in value at $\mathscr{E}''$ as a consequence of which the flow partially turns into the second sheet of a Riemannian surface.

To determine the shape of an ejection crater from Eqs. (2.2) and (2.4), we find the differential equation

$$i\frac{dz}{dt} = \frac{Ah^2}{r^2}\frac{(1-t^2)(1-t^2r^2)^2}{(1+t^2h^2)^4}. \tag{2.5}$$

We now determine A. We find from Eq. (2.5) that in a neighborhood of the point $t = ih$,

$$i(z - L) \approx \frac{Ah^2}{r^2}\frac{(1+h^2)(1+h^2r^2)^2}{(1-h^4)^4}(t - ih),$$

and from Eq. (2.1), that

$$w \approx -A\frac{1}{2(1+h^2)}\frac{1}{t-ih}.$$

Therefore, it follows that

$$w \approx \frac{A^2h^2}{2r^2}\frac{(1+h^2r^2)^2}{(1-h^4)^4}\frac{i}{z-L}.$$

Since $w \approx i/(z-L)$ in the neighborhood of B, we find that

$$A = \frac{\sqrt{2}\,r(1-h^4)^2}{h(1+h^2r^2)}$$

and we find from Eq. (2.5) that

$$i\frac{dz}{dt}=\frac{\sqrt{2}\,h(1-h^4)^2}{r(1+h^2r^2)}\frac{(1-t^2)(1-t^2r^2)^2}{(1+t^2h^2)^4}. \tag{2.6}$$

Integrating this equation between z = 0 and t = 0, we obtain

$$iz\frac{\sqrt{2}r(1+h^2r^2)}{(1-h^4)^2}=\frac{1}{3}\left[1+\frac{1+2r^2}{h^2}+\frac{r^4+2r^2}{h^4}+\frac{r^4}{h^6}\right]\frac{th}{(1+t^2h^2)^3}+$$
$$+\frac{1}{12}\left[5-\frac{1+2r^2}{h^2}-7\frac{r^4+2r^2}{h^4}-13\frac{r^4}{h^6}\right]\frac{th}{(1+t^2h^2)^2}+\frac{1}{8}\left[5-\frac{1+2r^2}{h^2}+\right.$$
$$\left.+\frac{r^4+2r^2}{h^4}+11\frac{r^4}{h^6}\right]\frac{th}{1+t^2h^2}-\frac{i}{16}\left[5-\frac{1+2r^2}{h^2}+\frac{r^4+2r^2}{h^4}-5\frac{r^4}{h^6}\right]\ln\frac{1+ith}{1-ith},$$

where r and h are connected by Eq. (2.3). When t = ih, we obtain a relation between the initial magnitude L and the parameter h. The graph of this dependence is depicted in Fig. 5 for $h < h_1$. When $t = e^{i\theta}$ $(0 < \theta < \pi/2)$ we obtain the ejection cone formula. A cone for $h = h_2$ and $L = L_2 \approx 0.3564$ is depicted in Fig. 3a.

2. $h > h_1$, r a complex number. In this case the exterior of a unit circle and a section along the imaginary semiaxis is the flow region in the $\omega$ plane (cf. Fig. 4b). The function $\omega$ has a second-order pole at B and nowhere vanishes in the flow region. If we continue $\omega$ to the entire complex plane according to symmetry, we find that

$$\omega=i\frac{(1+t^2h^2)^2}{(t^2+h^2)^2}. \tag{2.7}$$

Using Eq. (2.2), we obtain a differential equation for determining the shape of the ejection cone,

$$i\frac{dz}{dt}=-A\frac{h^2}{r^2}\frac{(1-t^2)(t^2-r^2)(1-t^2r^2)}{(1+t^2h^2)^4}.$$

When t = r $dz/dt \approx (t-r)$, so that the conformal property is violated at this point and $z-z_r \approx (t-r)^2$. The independent variable varies by $\pi$ as the point F is circuited in the t plane. Consequently, the independent variable will vary by $2\pi$ in a circuit in the z plane. Using the previously described method we find that

$$A=\frac{(1-h^4)(1+h^2)}{h},$$

and, using Eq. (2.3), we obtain

$$i\frac{dz}{dt}=h(1-h^4)(1+h^2)\frac{(1-t^2)\left[t^4+1-t^2\left[\left(\frac{1}{h}-h\right)^2-2\right]\right]}{(1+t^2h^2)^4}. \tag{2.8}$$

Integration of this equation yields

$$iz=\frac{(1-h^4)^3}{3h^6}\frac{th}{(1+t^2h^2)^3}-\frac{(1-h^4)^2(5+h^4)}{6h^6}\frac{th}{(1+t^2h^2)^2}+$$
$$+\frac{(1-h^4)(1+h^2)(3-h^2+h^4+h^6)}{4h^6}\frac{th}{1+t^2h^2}+i\frac{(1+h^2)^2(1-h^4)^2}{8h^6}\ln\frac{1+ith}{1-ith}. \tag{2.9}$$

When t = ih, we obtain the relation between L and the parameter h:

$$L=\frac{4}{3}+\frac{\left(\frac{1}{h}-h\right)^2\left[\left(\frac{1}{h}+h\right)^2+2\right]}{4}-\frac{\left(\frac{1}{h}-h\right)^2\left(\frac{1}{h}+h\right)^4}{8}\ln\frac{1+h^2}{1-h^2},$$

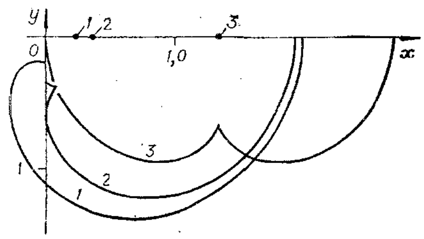


Fig. 6

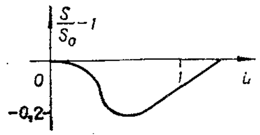


Fig. 7

whose graph is depicted in Fig. 5 for $h > h_1$. Substituting $t = i$, we obtain an equation for calculating the width of the ejection cone,

$$x_0 = \frac{(1+h^2)^3}{4h^3}\left[1 - \frac{2}{3}h^2 + h^4 - \frac{(1-h^2)^2(1+h^2)}{2h}\ln\frac{1+h}{1-h}\right].$$

When $h = h_1$, Eq. (2.6) coincides with Eq. (2.8), the independent variable of the complex velocity varies irregularly from $\pi/2$ to $3\pi/2$ at the point D (cf. 4a and b) and the ejection cone has the form given in Fig. 6 (curve 1). Evidently, part of the flow region is on the second sheet of a Riemannian surface. A further increase in h leads to the point F ($t = r$) in the $\omega$ plane (cf. Fig. 4b) moving along a circle and the independent variable of the complex velocity continuously varying from $3\pi/2$ to $\pi/2$ at this point. The velocity direction varies from $-3\pi/2$ to $-\pi/2$, part of the flow region on the second sheet of a Riemannian surface decreases, and, beginning with $h = h_3$, the entire flow fits on a single sheet (cf. Fig. 6, curve 2).

As $h \to 1$ dipole interaction decreases and a cone corresponding to a single dipole is obtained within the right half-plane (cf. Fig. 6, curve 3). The variable $h_3$ is determined from the condition that the point at which velocity is directed vertically downward is on the imaginary axis, i.e., $h_3$ is a solution of the system of equations

$$\begin{cases} \mathrm{Re}\,\omega = 0, \\ x = 0, \end{cases}$$

where $\omega$ and x are determined by Eqs. (2.7) and (2.9) when $t = e^{i\theta}$, $\arg(r) < \theta < \pi/2$ (arc FC). Numerical solution of the system yields $h_3 \approx 0.4933$.

Let us turn to Fig. 5. It follows from the solution of the problem that there exists a single-sheeted solution $h \le h_2$ and $h \ge h_3$, and that the solution is two-sheeted when $h_2 < h < h_3$. When $L_1 \le L \le L_2$ ($L_1 \approx 0.243$, $L_2 \approx 0.3564$) three values of the parameter correspond to the initial L, i.e., the problem has three solutions within this interval. Consequently, a one-sheeted solution may be correlated to every value of L. There exists two one-sheeted solutions when $L_3 \le L \le L_2$ ($L_3 \approx 0.3516$).

It should be noted that it is possible to construct one more solution when $0 \le h \le h_3$ that is everywhere one-sheeted but possesses a dead zone [5].* For this purpose it is necessary that the imaginary axis be a solid wall. When $h = h_3$, the cone touches this wall and a subsequent decrease in h leads to the formulation of a rectilinear vertical segment of the cone boundary on which flow rate first increases to some (determined in the course of solving the problem) $V_1$ and then again decreases to one. The flow region in the w plane remains invariant. A groove in the positive direction of the imaginary axis up to $V_1$ ($V_1 > 1$) appears in the $\omega$ plane when $h < h_3$. A dead zone that vanishes as $h \to 0$ is formed in the physical plane within the flow region.

The areas of ejection cones were calculated using the solution as a function of the distance between the charges ($S_0$ is the cone area at $L = 0$) (Fig. 7). Only one-sheeted solutions were considered. It is evident from the graph that interaction of the charges in this case as well does not increase the cone area. Maximal efficiency of charges is obtained when they do not interact and are set off together.

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